Eng Int Syst (2018) 2-3: 49–56

Engineering
Intelligent Systems

An integrated approach for reliability prediction of hydraulic system based on grey system theory and GO methodology

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The reliability of hydraulic systems is a key factor to ensure the safe and stable operation of equipment. It is very important to predict the reliability of the hydraulic system. Without considering the changes of reliability and failure rate during running time, the traditional method for reliability prediction cannot easily reflect the impact of processes under different operational conditions on reliability. To address this issue, this paper presents an integrated approach to predict the reliability of hydraulic systems for industrial applications. We do this by combining grey system theory and GO methodology. In order to validate the proposed approach, the grey model GM(1,1) is tested on practice data from industrial production. The GO model can then be established to predict the reliability of the hydraulic system by considering the various failure rates. Eventually, a case of a hydraulic system in a machining center is used to illustrate the method. The result demonstrates that the GO chart with grey dynamic prediction can be used to predict the reliability of hydraulic system successfully and more precisely.

Keywords: Grey system theory, GO methodology, Hydraulic system, Reliability prediction

1. INTRODUCTION

The reliability property of a device (i.e. a component or system) relates to its ability to perform its required function for the period of time it is required. Hydraulic system is the kernel of control and power transmission in many complex mechanical systems. It is worth noting that hydraulic system faults can result in great losses. (Chen, et. al., 2013; Rahimdel, et. al., 2013; Li, et. al., 2016). It is very important to predict the reliability of complex hydraulic systems for these and other reasons. However, traditional static reliability prediction methods cannot meet the needs of reliability analysis for increasingly complex hydraulic systems. The defects of functional units in the failure analysis process lead to uncertain problems, such as changeable working conditions of the equipment (the equipment is subject to environmental stress, random disturbances, and time stress factors), indeterminate factors on the failure mechanism of the action, discrete failure data obtained from experiments or in other ways, inaccurate quantification of the failure probability distribution due to abstract systems and coupling factors among practical engineering applications. The acquisition of hydraulic system reliability data requires a large number of reliability experiments. However, taking into account the cost, experimental cycle and operability, it is very difficult to obtain data to this extent.

In recent years, both numerical methods and intelligent systems are developed for reliability analysis. One of the most universally used methods is fuzzy theory (Knezevic and Odoom, 2001; Pillay and Wang, 2003; Rao and Dhingra, 1992; Ravi, et. al., 2000; Yadav, et. al., 2003). The significance of fuzzy variables are that they facilitate gradual transition between states and consequently, possess a natural capability to express and deal with observation and measurement uncertainties. However, when it comes to prediction, fuzzy variables do not have this ability. Grey system theory, first introduced by Deng (1989) in the early 1980s, is used to cope with systems that provide partial information or have a dynamic model. Based on the grey system theory, the first-order single variable grey model GM(1,1) has been widely applied to predictions of complex systems, such as vehicle fatality risk (Mao and Chirwa, 2006), Lorenz chaotic system (Zhang, et. al., 2009), labor formation (Yin and Tang, 2013) and the moving path of the typhoon (Chen and Huang, 2013). The GO method (Shen, et. al., 2000; Shen and Huang, 2004), which is capable of evaluating system reliability and availability, has been extensively employed in industrial fields, such as aircraft carriers, nuclear energy plants, petrochemical plants, and the like (Shen, et. al., 2006). For the GO method, the operators represent components or logical relationships in a system, which can be connected by the signal-flows. In this regard, GO model could be established smoothly by the system schematic diagram. In addition, the GO model can also provide rich reliability information through the modeling process. That is, all signal flow probabilities in each state leading to system success or failure, probabilities. In this paper, the GO method is used to establish the reliability model for a hydraulic system in the machining center. An integrated model based on the grey model GM(1,1) has been proposed to predict the reliability of this hydraulic system.

This paper is organized as follows. In section 2, grey system theory and grey model GM(1,1) are introduced. A new reliability prediction integrated model is established in section 3 based on grey system theory and GO methodology, namely the GGO model. In section 4, a real-life case analysis for a hydraulic system of tray automatic exchange device (TAED) in the machining center is organized explicitly to illustrate how this model works. Finally, section 5 summarizes the findings and addresses possible future research directions.

2. GREY SYSTEM THEORY AND GM(1,1)

The Grey system theory demonstrates the optimal and unique ability of performing fitting predictions using small data sets and limited information to allow fast, concise, accurate, and effective predictions and understand future trends, so it has become a preferred method to study and model systems in which the structure or operation mechanism is not completely known (Xiao, et. al., 2014; Mao, et. al., 2016). According to the uncertainties of failures and operation in a complex hydraulic system, it can be regarded as a grey system. The principle of grey system theory for failure prediction is using knowable information to infer the characteristics, status and trends of unknowable information with failure modes, and to make prediction and decision-making for the future development of failures in the system. This process is called whitenization.

2.1 Operation Rules of Grey System Theory

According to the theory, the unknown parameters of the system are represented by discrete or continuous grey numbers encoded by the symbol $\otimes$. The theory introduces a number of properties and operations on the grey numbers such as the core of the number $\hat{\otimes}$, its degree of greyness g°, and the whitenization of the grey number. The latter operation generally describes the preference of the number towards the range of its possible values (Bezuglov and Comert, 2016). The grey numbers that include both the upper limit and lower limit ($\otimes \in [a, b]$) is termed the interval grey number. Denoting grey numbers $\otimes_1 \in [a, b]$, $a < b$; $\otimes_2 \in [c, d]$, $c < d$, and $*$ is the operation between interval grey numbers. Then there is also the interval grey number $\otimes_3 = \otimes_1 * \otimes_2$, $\otimes_3 \in [e, f]$, $e < f$.

(1) If $\otimes_1 \in [a, b]$, $a < b$, and $\otimes_2 \in [c, d]$, $c < d$, it follows that:

$$\otimes_1 + \otimes_2 \in [a + c, b + d] \tag{1}$$

(2) If $\otimes \in [a, b]$, $a < b$, it follows that:

$$-\otimes \in [-b, -a] \tag{2}$$

(3) If $\otimes_1 \in [a, b]$, $a < b$, and $\otimes_2 \in [c, d]$, $c < d$, it follows that:

$$\otimes_1 - \otimes_2 = \otimes_1 + (-\otimes_2) \in a - d, b - c \tag{3}$$

(4) If $\otimes \in [a, b]$, $a < b$, and $a \neq 0$, $b \neq 0$, $ab > 0$, it follows that:

$$\otimes^{-1} \in \frac{1}{b}, \frac{1}{a} \tag{4}$$

(5) If $\otimes_1 \in [a, b]$, $a < b$, and $\otimes_2 \in [c, d]$, $c < d$, it follows that:

$$\otimes_1 \times \otimes_2 \in \min\{ac, ad, bc, bd\}, \max\{ac, ad, bc, bd\} \tag{5}$$

(6) If $\otimes_1 \in [a, b]$, $a < b$, $\otimes_2 \in [c, d]$, $c < d$, and $c \neq 0$, $d \neq 0$, $cd > 0$, it follows that:

$$\otimes_1 \div \otimes_2 \in \min\{\tfrac{a}{c}, \tfrac{a}{d}, \tfrac{b}{c}, \tfrac{b}{d}\}, \max\{\tfrac{a}{c}, \tfrac{a}{d}, \tfrac{b}{c}, \tfrac{b}{d}\} \tag{6}$$

2.2 Mathematical Model of GM(1,1)

The Grey model GM(1,1) is mainly used for the fitting and predicting of the eigenvalues of a dominant factor in complex systems. This is in order to reveal the variation rule of the dominant factor and the changes of state in the future. The original data possesses grey uncertainties, therefore grey model can solve many problems which are difficult for other prediction models to solve (Wang, et. al., 2009). In grey system theory, the accumulated generating operation (AGO) technique is applied to reduce the randomization of the raw data. This processed data becomes a monotonic increase sequence which complies with the solution of first order linear ordinary differential equation. Therefore, the solution curve would fit into the raw data with high precision. In the following section, the derivation of GM(1,1) is briefly described:

Step 1: Assume that the original series of data with n entries is

$$x^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(k), \cdots, x^{(0)}(n)\right\}, \quad (7)$$

where raw material $x^{(0)}$ stands for the non-negative original historical time series data.

Step 2: Construct $x^{(1)}$ by on time accumulated generating operation (1-AGO), which is

$$x^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(k), \cdots, x^{(1)}(n)\right\}, \quad (8)$$

where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, 3, \cdots, n$.

The mean generating neighbor series of $x^{(1)}$ is

$$z^{(1)} = \left\{z^{(1)}(2), z^{(1)}(3), \cdots, z^{(1)}(k), \cdots, z^{(1)}(n)\right\}, \quad (9)$$

where $z^{(1)}(k) = \left[x^{(1)}(k) + x^{(1)}(k-1)\right]/2, \quad k = 2, 3, \cdots, n$.

Step 3: The result of 1-AGO is a monotonic increase sequence which is similar to the solution curve of first order linear differential equation. As a result, the solution curve of following differential equation represents the approximation of 1-AGO data

$$\frac{d\hat{x}^{(1)}}{dt} + a\hat{x}^{(1)} = b, \quad (10)$$

where $\wedge$ represents grey predicted value complemented the corresponding initial condition, $\hat{x}^{(1)}(1) = x^{(0)}(1)$, with the model parameters a and b.

Step 4: The model parameters a and b can be solved by discretization of Eq. (10), and then we can deduce the equation

$$\frac{d\hat{x}^{(1)}}{dt} = \lim_{\Delta t \to 1} \frac{\hat{x}^{(1)}(t + \Delta t) - \hat{x}^{(1)}(t)}{\Delta t}, \quad (11)$$

If the sampling time interval is unity, then let $\Delta t \to 1$, and therefore the Eq. (11) reduces to

$$\frac{d\hat{x}^{(1)}}{dt} \cong x^{(1)}(k+1) - x^{(1)}(k) = x^{(0)}(k+1), \quad (12)$$

where $k = 1, 2, 3, \cdots$

The predicted value $\hat{x}^{(1)}$, background value, is defined as

$$\hat{x}^{(1)} \cong Px^{(1)}(k) + (1-P)x^{(1)}(k+1) = z^{(1)}(k+1), \quad (13)$$

where $k = 1, 2, 3, \cdots$

Here P is traditionally set to 0.5 in the original model. And the source model can be obtained

$$x^{(0)}(k) + az^{(1)}(k) = b, \quad (14)$$

where $k = 1, 2, 3, \cdots$

To do this, using the least square method and the Eq.(14), the model parameters a and b can be written as

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_N, \quad (15)$$

where B and Y_N are defined as follows

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, Y_N = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \quad (16)$$

Taking another point of view, due to the expansion of Eq. (15), the model parameters a and b are also expressed by the following parametric forms

$$a = \frac{CD - (n-1)E}{(n-1)F - C^2}, b = \frac{DF - CE}{(n-1)F - C^2}, \quad (17)$$

where C, D, E, and F are given by

$$\begin{aligned} C &= \textstyle\sum_{k=2}^{n} z^{(1)}(k), D = \sum_{k=2}^{n} x^{(0)}(k), \\ E &= \textstyle\sum_{k=2}^{n} z^{(1)}(k)x^{(0)}(k), F = \sum_{k=2}^{n} \left[z^{(1)}(k)\right]^2, \end{aligned} \quad (18)$$

Step 5: To solve Eq. (10) together with the initial condition, the particular solution is

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} + \frac{b}{a}, \quad (19)$$

where $k = 2, 3, 4, \cdots$

Hence, the desired prediction output at k step can be estimated by inverse accumulated generating operation (1-IAGO) which is defined as

$$\begin{aligned} &\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \\ &= (1 - e^{a})\left(x^{(0)}(1) - \tfrac{b}{a}\right)e^{-ak}, \end{aligned} \quad (20)$$

where $k = 1, 2, 3, \cdots$

2.3 Error Analysis

In order to determine the prediction precision of grey model GM(1,1), the posterior error inspection act can be employed to test it. If the prediction precision is not good, a residual grey model GM(1,1) is needed to establish to correct the original model.

Denoting the knowable data at time k as $x^{(0)}(k)$, the calculated value is $\hat{x}^{(0)}(k)$, so the mean original data is

$$\bar{x}^{(0)} = \frac{1}{n}\sum_{k=1}^{n} x^{(0)}(k); \quad (21)$$

the absolute residual is

$$q(k) = x^{(0)}(k) - \hat{x}^{(0)}(k); \quad (22)$$

the relative residual is

$$\varepsilon(k) = \frac{q(k)}{x^{(0)}(k)} \times 100\%; \quad (23)$$

the variance of original data is

$$S_1^2 = \frac{1}{n}\sum_{k=1}^{n}\left[x^{(0)}(k) - \bar{x}^{?0?}\right]^2; \quad (24)$$

the mean relative residual is

$$\bar{\varepsilon} = \frac{1}{n-1} \sum_{k=1}^{n} \varepsilon(k); \tag{25}$$

the residual variance is

$$S_2^2 = \frac{1}{n} \sum_{k=1}^{n} [\varepsilon(k) - \bar{\varepsilon}]^2; \tag{26}$$

the small error probability is

$$p = p\left\{|\varepsilon(k) - \bar{\varepsilon}| \leq 0.674 S_1\right\}; \tag{27}$$

the posterior error ratio is

$$c = S_2/S_1. \tag{28}$$

Small error probability and the posterior error ratio can be used to judge the failure prediction. That is, whether the results meet the precision requirements. If the residual model starts fitting at the mth residual, then the corrected residual model is

$$\begin{cases} \hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a}, k < m \\ \hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a} + \varepsilon^{(1)}(m), k = m \\ \hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a} + \left[\varepsilon^{(1)}(m) - \frac{b_1}{a_1}\right] \\ \left(e^{-a_1(k-m+1)} - e^{-a_1(k-m)}\right), k > m \end{cases} \tag{29}$$

3. RELIABILITY PREDICTION WITH GGO

Traditional reliability prediction methods always ignore the relation between the parts and entire system, that is, only to predict the entire hydraulic system with little consideration of reliability prediction of parts in the system or vice versa. Furthermore, the actual operation is a random process, and it is unreasonable that the probability of failure in this process is considered to be a precise value. For these defects, a GGO model, which combines the virtue of grey system theory and GO methodology, is established for reliability prediction of a complex hydraulic system.

3.1 GO Methodology

The GO methodology is a method of system reliability with success-oriented. The GO model is composed of operators and signal-flows, and the operators represent components or logical relationships in the system and the signals represent connections between the components. In the GO model, there are 17 kinds of operators (Fan, et. al., 2015) which are shown in Figure 1. All operators, which have its individual function states and computational rule, can generally be divided into three groups: logical operators, functional operators and special operators.

(1) Logical operators (including type 2, 9, 10, 11, 14, 15) represent just the specific computation logics without considering their own functional states.

(2) Functional operators (including type 1, 3, 4, 5, 6, 7, 8, 16, 17) have their own functional states as well as they have their various computation logic, and that is different from the logical operators.

(3) Special operators (including type 12 and 13): type 12, which is called a path separator, always represent a monopole multi-throw switch. Type 13 represents the multiple inputs and output component, which allows customization of existing parameters to achieve the function you need.

In the GO methodology, the signal is tagged by a non-negative integer, which represent the signal state, such as 0 which represents a premature state; $1, 2, \ldots, N-1$ represent multiple success states and N represents the failure state. The probability of input signal state and output signal state are represented, respectively, by PS(i) and PR(i), $i = 0, 1, 2, \ldots, N$. For the time-sequential system, $i = 0, 1, 2, \ldots, N$ can also represent the signal arrival time point. Among all operators, type 1, 2, 5, 6 and 10 are the most commonly used operators in GO model. Liu, et. al. (2015) present the quantification formulas of the operators.

3.2 Modelling of GGO

The process of modeling GGO to predict the reliability of the hydraulic system includes the following steps.

Step 1: Analyzing the given hydraulic system, specifying the range, features and components of the system, determining the structure and reliability index of the system, and drawing schematic drawing or flowchart of the system.

Step 2: Determining the input and output of the hydraulic system. The input comes from the external events of the system, and the output is a set of output signals which indicates the state of the system.

Step 3: Determining the normal operating state of the hydraulic system, confirming the minimum required set of output signals of normal operation system.

Step 4: The components in the hydraulic system are represented by operators, leading to a translation of into a schematic drawing or flowcharts of the system. These are then transformed into a GO chart, connecting the operators with signal flow.

Step 5: Translating the statistic site failure data into state probability data of all components in the hydraulic system, and then brought to the GM(1,1) prediction model. If the result of failure prediction does not meet the precision requirements, a residual grey model GM(1,1) could be established to correct the original model. Meanwhile, inputting the corrected data according to operator number can be done.

Step 6: According to the GO chart and grey prediction data from previous step, calculating the output signal of the hydraulic system gradually based on the operation rules of operator.

Step 7: Comparing the prediction precision of GGO prediction model with that of traditional static GO methodology and FTA.

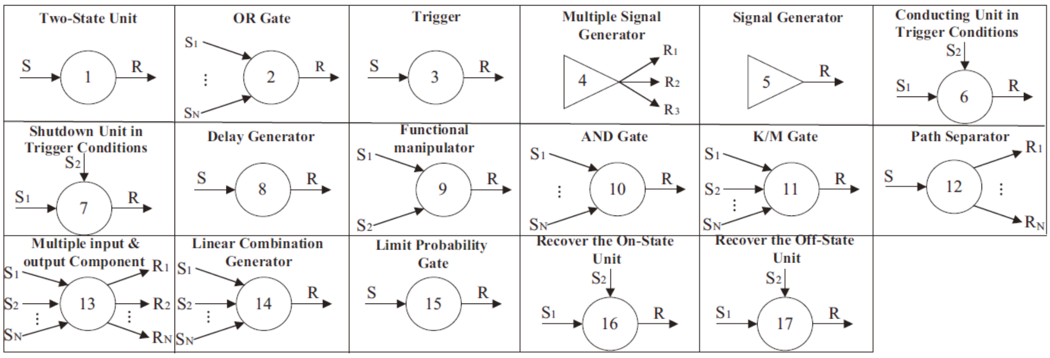
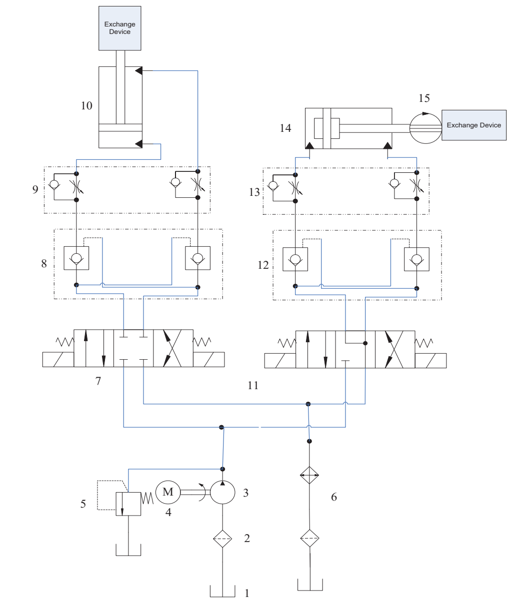


Figure 1 Operators defined in the GO methodology.

4. NUMERICAL EXAMPLE

The hydraulic system in the TAED is one of the most important functional units of the machining center, and it is also one of the parts with the higher failure rate. Consequently, it is very important to identify weaknesses and improve the efficiency of the machining center by the reliability prediction analysis.



1. Tank 2. Filter 3. Vane Pump 4. Motor 5. Relief Valve 6. Cooling Apparatus 7. Electromagnetic Reversing Valve 8. One-Way Valve 9. Throttle Valve 10. Hydraulic Cylinder 11. Electromagnetic Reversing Valve(A-P-T) 12. One-Way Valve 13. Throttle Valve 14. Hydraulic Cylinder 15. Gear Rack Pair

Figure 2 Systematic chart of hydraulic system in TAED.

4.1 Working principle of the TAED

A typical structure of the hydraulic system of the TAED is shown in Figure 2. First of all, the left end of the electromagnet of valve 7 is energized, which connected to the left side of the electromagnetic valve; at this moment, the lifting hydraulic system began to work; the cylinder lifted the bracket until the sensor switch detected the rising position, then the sensor switch sends electric signals to valve 7, which is out of power supplement under the control of electric device; now, the valve 7 is in middle and the bracket rose to the right position. Secondly, the moment the sensor switch detected the rising position, it sends signals to the control system, and the left end of the valve 11 is energized, which connects to the left side of the electromagnetic valve. At this moment, the rotary hydraulic system begins to work; the cylinder rotated the bracket until the sensor switch detected the bracket rotation of 180°, then the sensor switch send electric signals to valve 11, which is out of power supplement under the control of electric device. Now, the valve 11 is in middle and the bracket rotated to the right position. Thus, the exchange process of TAED is completed.

4.2 Modelling of TAED Based on GO Methodology

Based on the basic theory of the GO methodology, we can obtain the GO chart translated from the systematic diagram of hydraulic system of TAED, shown in Figure 3.

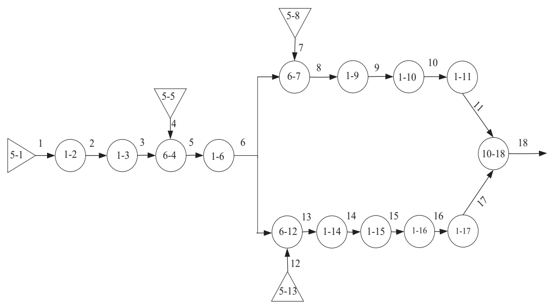


Figure 3 GO chart of hydraulic system in TAED.

4.3 Calculating the State Probability Based on GM(1,1)

Fifteen machining centers are chosen as a sample, and the number and the time of abnormal incidents in the hydraulic system of TAED is recorded at equal time intervals T=24h. The failure rate can be obtained according to the ratio of the number of abnormal units and total units during the interval. The state probability of each signal flow is obtained in each time interval T1-T5 based on the calculation rules. This is

Table 1 Name of table Operator data and predictive state probability of hydraulic system in TAED.

No.	Type of operator	Component	State probability					Predictive state probability
			T_1	T_2	T_3	T_4	T_5	T_6
1	5	Hydraulic oil tank	0.999	0.998	0.998	0.999	0.998	0.9974
2	1	Filter	0.995	0.994	0.993	0.994	0.994	0.9929
3	1	Cooling apparatus	0.999	0.998	0.997	0.999	0.999	0.9948
4	6	Vane pump	0.990	0.986	0.986	0.990	0.987	0.9824
5	5	Motor	0.997	0.997	0.996	0.997	0.997	0.9961
6	1	Relief valve	0.994	0.992	0.992	0.994	0.993	0.9893
7	6	Electromagnetic Reversing Valve	0.993	0.991	0.990	0.993	0.992	0.9933
8	5	Control signal	0.999	0.998	0.998	0.999	0.998	0.9976
9	1	One-Way Valve	0.995	0.993	0.993	0.994	0.994	0.9907
10	1	Throttle Valve	0.995	0.993	0.993	0.994	0.993	0.9925
11	1	Hydraulic Cylinder	0.993	0.992	0.992	0.993	0.993	0.9922
12	6	Electromagnetic Reversing Valve	0.993	0.992	0.991	0.993	0.992	0.9916
13	5	Control signal	0.999	0.998	0.998	0.999	0.998	0.9983
14	1	One-Way Valve	0.995	0.995	0.994	0.995	0.995	0.9946
15	1	Throttle Valve	0.995	0.994	0.994	0.995	0.994	0.9943
16	1	Hydraulic Cylinder	0.993	0.991	0.991	0.993	0.992	0.9915
17	1	Gear Rack Pair	0.993	0.991	0.991	0.992	0.992	0.9897

Table 2 Precision examine

No.	Type of operator	Component	Precision examine coefficient	
			p	c
1	5	Hydraulic oil tank	0.9624	0.3164
2	1	Filter	0.9638	0.3066
3	1	Cooling apparatus	0.9513	0.2216
4	6	Vane pump	0.9607	0.1758
5	5	Motor	0.9662	0.2863
6	1	Relief valve	0.9505	0.3327
7	6	Electromagnetic Reversing Valve	0.9606	0.3094
8	5	Control signal	0.9582	0.2125
9	1	One-Way Valve	0.9517	0.2036
10	1	Throttle Valve	0.9531	0.1865
11	1	Hydraulic Cylinder	0.9511	0.2986
12	6	Electromagnetic Reversing Valve	0.9606	0.3411
13	5	Control signal	0.9581	0.2358
14	1	One-Way Valve	0.9520	0.2170
15	1	Throttle Valve	0.9575	0.2061
16	1	Hydraulic Cylinder	0.9605	0.2653
17	1	Gear Rack Pair	0.9522	0.3022

shown in Table 1. Then five state probability of each component in the hydraulic system of TAED is regarded as the original data and brought into Eq. (10), Eq. (19), Eq. (20), the probability of each component in the next interval T6 is obtained, such as shown in the predictive state probability column of Table 1. Then, the state probability and predictive state probability of each component operator is brought into Eq. (27) and Eq. (28) to implement posterior error inspection act, such as shown in Table 2. Comparing with the indices in Table 3, it shows that the prediction results meet the precision requirements, and the prediction model is credible.

Table 3 Grade of precision examine.

Grade	p	c
Excellent	> 0.95	< 0.32
Good	> 0.80	< 0.45
Average	> 0.70	< 0.55
Bad	≤ 0.70	≥ 0.65

4.4 Calculating Reliability Using GO Methodology

In this hydraulic system of TAED, the signal flow 6 is named the shared signal for its being separated into two ways to cross the electromagnetic valve. The failure probability of the hydraulic system with signal flows cannot be calculated directly. Therefore we have to reference a new mathematical methodology to quantify the probability, and its quantitative calculating process is as follows. Suppose that P_{si} stands for the state probability of the signal flow of number i, and P_{ci} stands for the state probability of the operator of number i, then

(1) the input operator:

$$P_{S1} = P_{C1}, P_{S4} = P_{C5}, P_{S7} = P_{C8}, P_{S12} = P_{C13};$$

(2) the signal flow 6:

$$P_{S6}{=}P_{C1}P_{C2}P_{C3}P_{C4}P_{C5}P_{C6};$$

(3) the signal flow 11:

$$P_{S11}{=}P_{S6}P_{C7}P_{C8}P_{C9}P_{C10}P_{C11};$$

(4) the signal flow 17:

$$P_{S17}{=}P_{S6}P_{C12}P_{C13}P_{C14}P_{C15}P_{C16}P_{C17}.$$

Let P_{s6}=0, then we can get the state probability of signal flow 18:

$$P^0_{S18}{=}P_{S11}P_{S17};$$

Let P_{s6}=1, then we can get the state probability of signal flow 18:

$$P^1_{S18}{=}P_{S11}P_{S17} = P_{C7}P_{C8}P_{C9}P_{C10} \\ P_{C11}P_{C12}P_{C13}P_{C14}P_{C15}P_{C16}P_{C17}.$$

Finally, we can get the accurate state probability of signal flow 18:

$$P_{S18} = (1 - P_{S6})\, P^0_{S18} + P_{S6}P^1_{S18}.$$

The signal flow 18 represents the output signal of the hydraulic system, so the performance of the hydraulic system can be assessed according to the state probability of signal flow 18 and the functional requirements of the hydraulic system. The predictive state probability of each component in Table 1 is brought into the quantitative calculating process above, so we can obtain the state probability of the TAED hydraulic system in the machining center. That is, as $P(t) = 0.8857$.

4.5 Benchmark Analysis

In order to verify the calculation precision of the GGO model, we used the traditional GO methodology and FTA to calculate the reliability of the hydraulic system of TAED mentioned above as well. Due to the limited space, the FTA process has not been elaborated here. The calculation results are shown in Table 4. From the benchmark analysis results shown in Table 4 above, we can see clearly that comparing with GO methodology and FTA, the relative error of the GGO model proposed in this paper is the smallest one. Meanwhile, GGO model can reflect clearly that the functional and logical relationship between system and components, so it is very adaptive to the reliability prediction of a complex hydraulic system.

Table 4 Benchmark analysis.

Reliability probability	The authentic reliability probability	The analytical reliability probability		
		GGO	GO	FTA
	0.8906	0.8857	0.8527	0.8649
Relative error	0	0.55%	4.26%	2.89%

5. CONCLUSION

In this paper, the reliability of a complex hydraulic system is analysed, and the new reliability prediction integrated model based on grey system theory and GO methodology is established. The improvement of the calculation of GO methodology based on the application of grey system theory makes up for the lack of authenticity of the failure data sources, and it resolves the uncertainty on the failure rate of components as well. The precision of the GGO model is proved by being introduced to reliability prediction of the hydraulic system of TAED in the machining center. It is compared with the traditional GO methodology, FTA and the authentic failure data, so it provides a comprehensive basis for reliability design and distribution. However, under the limited conditions, the failure rate of components is lack of certain accuracy, which is the original of the error. So in the future, it is necessary to use more accurate methods to collect more data.

6. ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China (No. 31601210), and the Fundamental Research Funds for the Central Universities (No. SWU114038, No. XDJK2015B008).

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